Modeling Calibration Standards

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Symbols:

System impedance	Z_0
Frequency	f
Length of transmission line	ℓ
Propagation coefficient	γ
Offset delay	τ_{offs}
Offset loss	L_{offs}
Offset impedance	Z_{offs}

The one-port calibration standards are modeled as follows: The port of the standard is connected to a transmission line of length ℓ, which is terminated with a frequency dependent impedance Z_{T}. The transmission line is exhaustively described by the parameters Z_{offs}, τ_{offs}, and L_{offs}. In fitting the model parameters to the measured data we shall assume that $Z_{\text{offs}} = Z_0$. The derivation of this model from first principles as well as the assumptions and approximations that underlie the equations below are given in [1].

Impedance of the transmission line:

$$Z_{\text{c}} = \left(Z_{\text{offs}} + \frac{L_{\text{offs}}}{4\pi f} \sqrt{\frac{f}{f_0}} \right) - \mathrm{i} \frac{L_{\text{offs}}}{4\pi f} \sqrt{\frac{f}{f_0}}.$$

Product of propagation coefficient γ with the length ℓ of the line:

$$\gamma\ell = \frac{L_{\text{offs}}\tau_{\text{offs}}}{2Z_{\text{offs}}} \sqrt{\frac{f}{f_0}} + \mathrm{i} \left(2\pi f \tau_{\text{offs}} + \frac{L_{\text{offs}}\tau_{\text{offs}}}{2Z_{\text{offs}}} \sqrt{\frac{f}{f_0}} \right),$$

where $f_0 = 10^9\,\text{Hz}$.

Reflection coefficient corresponding to the impedance Z_{c} with respect to the system impedance Z_0:

$$\Gamma_1 = \frac{Z_{\text{c}} - Z_0}{Z_{\text{c}} + Z_0}.$$

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Reflection coefficient corresponding to the impedance Z_T terminating the line, with respect to the system impedance Z_0:

$$\Gamma_\mathrm{T} = \frac{Z_\mathrm{T} - Z_0}{Z_\mathrm{T} + Z_0}.$$

With these quantities the reflection coefficient at the port of the calibration standard (i. e., its S_{11}) can be written as:

$$\Gamma = \frac{\Gamma_1\bigl(1 - \mathrm{e}^{-2\gamma\ell} - \Gamma_1\Gamma_\mathrm{T}\bigr) + \mathrm{e}^{-2\gamma\ell}\Gamma_\mathrm{T}}{1 - \Gamma_1\bigl(\mathrm{e}^{-2\gamma\ell}\Gamma_1 + (1 - \mathrm{e}^{-2\gamma\ell})\Gamma_\mathrm{T}\bigr)}.$$

For the open standard one chooses:

$$Z_\mathrm{T} = \frac{1}{2\pi\mathrm{i}f \cdot C(f)} \quad \text{with } C(f) = C_0 + C_1 f + C_2 f^2 + C_3 f^3.$$

For the short standard one chooses:

$$Z_\mathrm{T} = 2\pi\mathrm{i}f \cdot L(f) \quad \text{with } L(f) = L_0 + L_1 f + L_2 f^2 + L_3 f^3.$$

References

[1] Keysight Technologies: *Specifying Calibration Standards and Kits for Keysight Vector Network Analyzers.* Application Note 5989-4840EN.